State both parts of the Fundamental Theorem of Calculus, and the Net Change Theorem. **SCORE: _____ / 10 PTS**

SEE QUIZ 3 KEY

What is the difference between using $f(x_i^*)$ and $f(a+i\Delta x)$ in the definition of the definite integral ? **SCORE: _____ / 10 PTS**

What must you know to be true about f before you can use $f(a+i\Delta x)$ in the definition ?

(Your answers may refer to the fact that the definite integral equals the area under a curve which is above the $x-$axis.)

$f(x_i^*)$ USES ANY POINT IN EACH SUBINTERVAL TO DETERMINE THE HEIGHT OF A RECTANGLE

$f(a+i\Delta x)$ USES AN ENDPOINT

f MUST BE CONTINUOUS TO USE $f(a+i\Delta x)$

The table below gives the rate $r(t)$ at which water is flowing out of a garden hose into a swimming pool (in gallons per minute), where t is the number of minutes since 3 pm. At 3 : 08 pm, there were 22 gallons of water in the swimming pool. **SCORE: _____ / 25 PTS**

t	0	2	4	6	8	10	12	14	16	18	20	22	24	26
$r(t)$	1	3	2	0	2	1	3	4	0	2	3	1	0	4

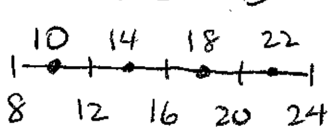
[a] Write an expression (involving an integral) for the amount of water in the pool at 3 : 24 pm.

$V(t)$ = VOLUME OF WATER IN POOL $\Longrightarrow V'(t) = r(t)$

$\int_8^{24} r(t)\,dt = V(24) - V(8)$ SO $V(24) = 22 + \int_8^{24} r(t)\,dt$

[b] Estimate the amount of water in the pool at 3 : 24 pm using the answer to part [a], 4 subintervals and the Midpoint Rule.

$V(24) \approx 22 + [r(10)\cdot 4 + r(14)\cdot 4 + r(18)\cdot 4 + r(22)\cdot 4]$



$= 22 + [1 + 4 + 2 + 1]\cdot 4$

$\Delta x = \frac{24-8}{4} = 4$

$= 22 + 32$

$= 54$ GALLONS

Evaluate the following integrals. **SCORE: _____ / 50 PTS**

[a] $\int_{-2}^{2} (2x^4 - 1)\tanh x \, dx$

$(2(-x)^4 - 1)\tanh(-x)$

$= (2x^4 - 1)(-\tanh x)$

$= -(2x^4 - 1)\tanh x$

INTEGRAND IS ODD
AND CONTINUOUS,
SO INTEGRAL $= 0$

[b] $\int \frac{1}{x \ln x} \, dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x} dx$

$\int \frac{1}{u} du = \ln|u| + C$

$= \ln|\ln x| + C$

[c] $\int_{-1}^{1} \frac{8x^4}{\sqrt[3]{6x^5 + 7}} \, dx$

$u = 6x^5 + 7 \longrightarrow x = -1 \Longrightarrow u = 1$

$x = 1 \Longrightarrow u = 13$

$\frac{du}{dx} = 30x^4$

$dx = \frac{1}{30x^4} du$

$\frac{8x^4}{\sqrt[3]{6x^5+7}} dx = \frac{\cancel{8}^{4}\cancel{x^4}}{\sqrt[3]{6x^5+7}} \cdot \frac{1}{\cancel{30}_{15}\cancel{x^4}} du$

$= \frac{4}{15} u^{-\frac{1}{3}} du$

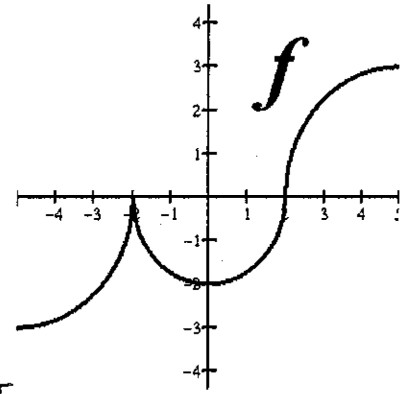
$\int_1^{13} \frac{4}{15} u^{-\frac{1}{3}} du$

$= \frac{\cancel{4}^{2}}{\cancel{15}_{5}} \cdot \frac{\cancel{3}}{2} u^{\frac{2}{3}} \Big|_1^{13}$

$= \frac{2}{5}(13^{\frac{2}{3}} - 1)$

Let $g(x) = \int_5^x f(t)\,dt$, where f is the function whose graph is shown on the right. **SCORE: _____ / 40 PTS**

<u>NOTE: The graph of *f* consists of a quarter circle of radius 3, followed by a semi-circle of radius 2, followed by another quarter circle of radius 3.</u>



[a] Find $g(-2)$.

$$\int_5^{-2} f(t)\,dt = -\int_{-2}^{5} f(t)\,dt$$

$$= -\left[\int_{-2}^{2} f(t)\,dt + \int_2^5 f(t)\,dt\right]$$

$$= -\left[-\tfrac{1}{2}\pi(2)^2 + \tfrac{1}{4}\pi(3)^2\right] = -\frac{\pi}{4}$$

[b] Find the x – coordinates of all inflection points of g. **<u>Explain your answer very briefly.</u>**

g HAS INFLECTION POINTS WHERE $f = g'$ CHANGES FROM INCREASING TO DECREASING OR DECREASING TO INCREASING

IE. AT $x = -2, 0$

[c] If $k(x) = \int_4^{x^3-8} f(t)\,dt$, find $k'(2)$. **<u>HINT: Find an algebraic expression for $k(x)$ first.</u>**

$$k'(x) = \frac{d}{dx}\int_4^{x^3-8} f(t)\,dt$$

$$= \frac{d}{d(x^3-8)}\int_4^{x^3-8} f(t)\,dt \cdot \frac{d(x^3-8)}{dx}$$

$$= f(x^3-8)\cdot 3x^2$$

$$k'(2) = f(0)\cdot 3(2)^2$$

$$= -2\cdot 12$$

$$= -24$$

Find $\frac{d}{dx}\sinh^{-1}(\operatorname{csch} x)$. Assume that $x > 0$. **SCORE: _____ / 15 PTS**

$$= \frac{1}{\sqrt{1+\operatorname{csch}^2 x}} \cdot -\operatorname{csch} x \coth x$$

$$= \frac{1}{\sqrt{\coth^2 x}} \cdot -\operatorname{csch} x \coth x$$

$$= \frac{1}{\coth x} \cdot -\operatorname{csch} x \coth x = -\operatorname{csch} x$$